CALCULUS I/MATH 150

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EXAM 2/CHAPTERS 2.2-2.6, 3.2

- π 50 POINTS POSSIBLE
- π YOUR WORK MUST SUPPORT YOUR ANSWER FOR FULL CREDIT TO BE AWARDED
- π TI-83/84/85/86 GRAPHING CALCULATOR IS PERMITTED
- π PROVIDE EXACT ANSWERS (NO DECIMALS PLEASE) UNLESS OTHERWISE INDICATED

ONCE YOU BEGIN THE EXAM, YOU MAY NOT LEAVE THE PROCTORING CENTER UNTIL YOU ARE FINISHED. THIS MEANS NO BATHROOM BREAKS…

NAME Key

MATH 150/GRACEY NAME___________________________________

EXAM 2/PART 2/CHAPTER 2.2-2.6, 3.2

50 POINTS POSSIBLE/BOX YOUR FINAL ANSWER

TI-83/84/85/86 GRAPHING CALCULATOR PERMITTED

FULL CREDIT WILL BE AWARDED BASED UPON WORK SHOWN—YOUR WORK MUST SUPPORT YOUR RESULTS

NO DECIMALS UNLESS OTHERWISE INDICATED

(25 POINTS) Problems 1-5. Find the derivative of the functions below with respect to the independent variable. Each item is worth 8 points. EXACT, FULLY SIMPLIFIED ANSWERS ONLY!!! This means a single rational expression which has NO COMPLEX FRACTIONS or negative powers.

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1. $\frac{d}{dx} f(x) = \frac{d}{dx}\left(4-x^6\right)^{25}$

$$f'(x) = 25\left(4-x^6\right)^{24}\frac{d}{dx}\left(4-x^6\right)$$

$$f'(x) = 25\left(4-x^6\right)^{24}\left(-6x^5\right)$$

$$\boxed{f'(x) = -150x^5\left(4-x^6\right)^{24}}$$

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2. $\frac{d}{dx} y = \frac{d}{dx}(x\cos 3x)$

$$\frac{dy}{dx} = \left(\frac{d}{dx}x\right)(\cos 3x) + (x)\left(\frac{d}{dx}\cos 3x\right)$$

$$\frac{dy}{dx} = 1(\cos 3x) + x\left[-\sin(3x)\frac{d}{dx}(3x)\right]$$

$$\frac{dy}{dx} = \cos 3x + x\left[(-\sin 3x)(3)\right]$$

$$\boxed{\frac{dy}{dx} = \cos 3x - 3x\sin 3x}$$

3. $\frac{d}{dt} h(t) = \frac{d}{dt}\left(\frac{1-t^{2/3}}{1+t^{2/3}}\right)$

$$h'(t) = \frac{\left[\frac{d}{dt}\left(1-t^{2/3}\right)\right]\left(1+t^{2/3}\right) - \left(1-t^{2/3}\right)\left[\frac{d}{dt}\left(1+t^{2/3}\right)\right]}{\left(1+t^{2/3}\right)^2}$$

$$h'(t) = \frac{-\frac{2}{3}t^{-1/3}\left(1+t^{2/3}\right) - \left(1-t^{2/3}\right)\left(\frac{2}{3}t^{-1/3}\right)}{\left(1+t^{2/3}\right)^2}$$

$$h'(t) = \frac{-\frac{2}{3}t^{-1/3}\left[\left(1+t^{2/3}\right) + \left(1-t^{2/3}\right)\right]}{\left(1+t^{2/3}\right)^2}$$

$$h'(t) = \frac{-\frac{2}{3}t^{-1/3}(2)}{\left(1+t^{2/3}\right)^2}$$

$$\boxed{h'(t) = \frac{-4}{3t^{1/3}\left(1+t^{2/3}\right)^2}}$$

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4. $f(x) = \frac{x^3+1}{x+1}$

easy way

$$f(x) = \frac{\cancel{(x+1)}(x^2-x+1)}{\cancel{x+1}}$$

$$\frac{d}{dx} f(x) = \frac{d}{dx}\left(x^2-x+1\right)$$

$$\boxed{f'(x) = 2x-1}$$

$$\begin{array}{r|rrrr} -1 & 2 & 3 & 0 & -1 \\ & & -2 & -1 & 1 \\ \hline & 2 & 1 & -1 & 0 \end{array}$$

hard way

$$\frac{d}{dx} f(x) = \frac{d}{dx}\left(\frac{x^3+1}{x+1}\right)$$

$$f'(x) = \frac{\left[\frac{d}{dx}\left(x^3+1\right)\right](x+1) - \left(x^3+1\right)\left[\frac{d}{dx}(x+1)\right]}{(x+1)^2}$$

$$f'(x) = \frac{(3x^2)(x+1) - (x^3+1)(1)}{(x+1)^2}$$

$$f'(x) = \frac{3x^3+3x^2-x^3-1}{(x+1)^2}$$

$$f'(x) = \frac{2x^3+3x^2-1}{(x+1)^2}$$

$$f'(x) = \frac{\cancel{(x+1)}(2x^2+x-1)}{(x+1)^{\cancel{2}}}$$

$$f'(x) = \frac{(2x-1)\cancel{(x+1)}}{\cancel{x+1}}$$

$$\boxed{f'(x) = 2x-1}$$

5. $f(\theta)=\left(\dfrac{\sin\theta}{\cos\theta}\right)^2$

easy way

$$\frac{d}{d\theta}f(\theta)=\frac{d}{d\theta}(\tan\theta)^2$$

$$f'(\theta)=2(\tan\theta)^1\frac{d}{d\theta}(\tan\theta)$$

$$f'(\theta)=(2\tan\theta)(\sec^2\theta)$$

$$\boxed{f'(\theta)=2\tan\theta\sec^2\theta}$$

hard way

$$f'(\theta)=2\left(\frac{\sin\theta}{\cos\theta}\right)^1\frac{d}{d\theta}\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$f'(\theta)=\frac{2\sin\theta}{\cos\theta}\left[\frac{\left[\frac{d}{d\theta}(\sin\theta)\right]\cos\theta-\sin\theta\left[\frac{d}{d\theta}(\cos\theta)\right]}{\cos^2\theta}\right]$$

$$f'(\theta)=\frac{2\sin\theta}{\cos^3\theta}(\cos\theta\cos\theta-\sin\theta(-\sin\theta))$$

$$f'(\theta)=\frac{2\sin\theta}{\cos^3\theta}(\cos^2\theta+\sin^2\theta)$$

$$f'(\theta)=\frac{2\sin\theta}{\cos^3\theta}(1)$$

$$f'(\theta)=2\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos^2\theta}$$

$$\boxed{f'(\theta)=2\tan\theta\sec^2\theta}$$

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6. (5 POINTS) Find $\dfrac{dy}{dx}$.

$$\frac{d}{dx}\left(x^2y-xy^2\right)=\frac{d}{dx}5$$

$$\frac{d}{dx}(x^2y)-\frac{d}{dx}(xy^2)=0$$

$$\left[\left(\frac{d}{dx}x^2\right)y+x^2\left(\frac{d}{dx}y\right)\right]-\left[\left(\frac{d}{dx}x\right)y^2+x\left(\frac{d}{dx}y^2\right)\right]=0$$

$$2xy+x^2\frac{dy}{dx}-\left(1y^2+x\left[2(y)^1\frac{d}{dx}(y)\right]\right)=0$$

$$2xy+x^2\frac{dy}{dx}-\left(y^2+2xy\frac{dy}{dx}\right)=0$$

$$2xy+x^2\frac{dy}{dx}-y^2-2xy\frac{dy}{dx}=0$$

$$\frac{dy}{dx}(x^2-2xy)=y^2-2xy$$

$$\frac{dy}{dx}=\frac{y^2-2xy}{x^2-2xy}$$

$$\boxed{\frac{dy}{dx}=\frac{y(y-2x)}{x(x-2y)}}$$

7. (5 POINTS) Find $\dfrac{d^2y}{dx^2}$.

$$y = \frac{5}{3x-7}$$

$$\frac{d}{dx}y = \frac{d}{dx}\left(5(3x-7)^{-1}\right)$$

$$\frac{dy}{dx} = 5\left[-1(3x-7)^{-2}\frac{d}{dx}(3x-7)\right]$$

$$\frac{dy}{dx} = 5\left(-(3x-7)^{-2}(3)\right)$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(-15(3x-7)^{-2}\right)$$

$$\frac{d^2y}{dx^2} = -15\left[-2(3x-7)^{-3}\frac{d}{dx}(3x-7)\right]$$

$$\rightarrow \frac{d^2y}{dx^2} = -15\left[-2(3x-7)^{-3}(3)\right]$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{90}{(3x-7)^3}}$$

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8. (5 points) Determine whether the Mean Value Theorem can be applied to $f(x) = \sqrt{x} + 16$ on the closed interval $[0,4]$. If so, find all values of c such that $f'(c) = \dfrac{f(b) - f(a)}{b-a}$.

Since f is continuous on $[0,4]$ and differentiable on $(0,4)$, the Mean Value Thm can be applied

(1)

$$\frac{d}{dx}(f(x)) = \frac{d}{dx}\left(x^{1/2} + 16\right)$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

(2)

$a = 0,\ b = 4$

$f(0) = \sqrt{0} + 16 = 16$

$f(4) = \sqrt{4} + 16 = 18$

$$\frac{f(b) - f(a)}{b-a} = \frac{f(4) - f(0)}{4-0} = \frac{18-16}{4} = \frac{1}{2}$$

(3)

$$\frac{1}{2\sqrt{x}} = \frac{1}{2} \rightarrow 2\sqrt{x} = 2$$

$\sqrt{x} = 1$

$x = \pm 1$

$\boxed{c = 1}$

9. (5 POINTS) Solve the word problem showing all steps.

A hot tub in the shape of a semi-sphere is draining at a rate of 2 meters cubed per minute. Find the instantaneous rate of change of the radius of the hot tub when the radius measures 3 meters. Please round to the nearest hundredth.

$$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$\frac{d}{dt}V = \frac{d}{dt}\left(\frac{2}{3}\pi r^3\right)$$

$$\frac{dV}{dt} = \frac{2\pi}{3}\left[3(r)^2\frac{d}{dt}(r)\right]$$

$$\frac{dV}{dt} = \frac{2\pi}{3}\left[3r^2\frac{dr}{dt}\right]$$

$$\frac{dV}{dt} = 2\pi r^2\frac{dr}{dt}$$

$$-2 = 2\pi(3)^2\frac{dr}{dt}$$

$$\frac{-1}{9\pi} = \frac{dr}{dt}$$

$$\frac{dr}{dt} \approx 0.04 \text{ m/min}$$

When the radius is 3 m, the hot tub the instaneous rate of change of the radius of the hot tub is approximately 0.04 m/min

$$\frac{dV}{dt} = -2 \text{ m}^3/\text{min}$$

want $\frac{dr}{dt}$ when $r=3$

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10. (5 POINTS) Find the equation of the line tangent to the graph of $f(x) = 8 + \sqrt[3]{x}$ at $x = 27$.

① $$\frac{d}{dx}f(x) = \frac{d}{dx}\left(8 + x^{1/3}\right)$$

$$f'(x) = \frac{1}{3}x^{-2/3}$$

$$f'(x) = \frac{1}{3\left(\sqrt[3]{x}\right)^2}$$

② $$f'(27) = \frac{1}{3\left(\sqrt[3]{27}\right)^2}$$

$$f'(27) = \frac{1}{3 \cdot 9}$$

$$f'(27) = \frac{1}{27}$$

③ $$f(27) = 8 + \sqrt[3]{27} = 11$$

④ $$\boxed{y - 11 = \frac{1}{27}(x - 27)}$$

or

$$\boxed{y = \frac{1}{27}x + 10}$$